A General Formulation for Connecting Sources and Passive Lumped-Circuit Elements Across Multiple 3-D FDTD Cells

Carl H. Durney, Wenquan Sui, Douglas A. Christensen, and Jingyi Zhu

Abstract—**A previous extension of the finite-difference time-domain (FDTD) method to include lumped-circuit elements is further extended to model lumped-element circuits connected across multiple FDTD cells. This formulation is needed to model many kinds of circuits, like those with a transistor or other active device connected across a transmission line with more than one dielectric. The FDTD analysis of a shielded suspended microstrip transmission line excited by a current source in parallel with a resistance illustrates the usefulness of the formulation.**

I. Introduction

IN [1], we extended the finite-difference time-domain (FDTD) method to include lumped-circuit elements (both sources and passive elements, linear and nonlinear) in single FDTD cells. Others have applied and extended this technique [2]–[4]. A method has also been described for incorporating SPICE lumped circuits as sub-grid models into the FDTD formulation [5]. Here we describe methods needed for FDTD analysis of structures excited across multiple FDTD cells containing different dielectrics [6], and of devices with lumped elements connected across multiple FDTD cells; for example, a microstrip transmission line with two dielectrics excited by either a voltage or current source. This present work thus allows modeling of sources and passive lumped-circuit elements across multiple FDTD cells, whereas our previous work modeled only sources and circuit elements connected across single FDTD cells.

II. Derivation of the Equations

Following well-known procedures, we derive the FDTD equations from the integral form of Maxwell's equations

$$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\int_S \mu \frac{\partial \boldsymbol{H}}{\partial t} \cdot d\boldsymbol{S} \tag{1}$$

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = \int_S \boldsymbol{J} \cdot d\boldsymbol{S} + \int_S \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} \cdot d\boldsymbol{S} \tag{2}$$

by evaluating the integrals on a standard Yee cell. Fig. 1 shows how to include a lumped-element model across more than one FDTD cell (three here). The key concepts required to connect lumped-element circuit models with electromagnetic-field models is the relationship of electric field to voltage and magnetic field to current [7]. In this context, the $\boldsymbol{H} \cdot d\boldsymbol{l}$ term on each leg of the contour of integration corresponds to a loop current circulating around the $\boldsymbol{H}$ according to the right-hand rule (for example, a loop current around $H_{z,i,j,k}$ along the path *mnopm* in Fig. 1). Also, the right-hand side of (2) is equal to the total convection and conduction current (first term) plus the displacement current (second term) through the surface S. These two currents must be equal to the total current, which is represented by the left-hand side. Therefore to include the lumped-element model into the FDTD equations, the current $-I_c$ in Fig. 1 is added to the left-hand side of (2) to give

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} - I_c = \int_S \boldsymbol{J} \cdot d\boldsymbol{S} + \int_S \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} \cdot d\boldsymbol{S}. \tag{3}$$

Because the lumped-element model is assumed to include all the effects of its spatial distribution (stray capacitance, inductance, etc.), it is considered to occupy zero volume in FDTD space.

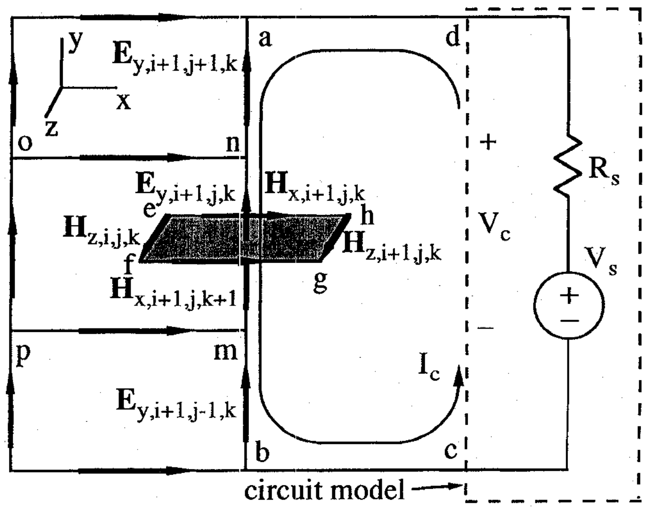

Fig. 1. Lumped-element circuit connected across multiple Yee cells (three in this example) and paths of integration for calculating one of the field components, $E_{y,i+1,j,k}$. The circuit example used in the text for deriving FDTD equations is a voltage source and series resistance, as shown in the dashed-line box. For clarity, one face each of only three of the Yee cells is shown.

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Using the lumped-element circuit shown in Fig. 1 as an example, we next develop the FDTD equations in four steps: 1) evaluate (1) following a path around the edges of the face of each Yee cell in the usual way; 2) evaluate (1) around a path across the lumped-element circuit ($abcda$ in Fig. 1); 3) relate V_c to I_c for the particular lumped-element circuit being modeled; and 4) evaluate (3) around paths like $efghe$ in Fig. 1. Because the first step gives the standard FDTD relations, it is not illustrated here. For the circuit in Fig. 1 (assuming the circuit occupies zero volume in FDTD space), the second step gives

$$V_c = -(E_{y,i+1,j+1,k} + E_{y,i+1,j,k} + E_{y,i+1,j-1,k})\delta y. \quad (4)$$

For the circuit in Fig. 1, the third step gives $I_c = (V_s - V_c)/R_s$ (similar relations may be derived for other circuits). Substituting (4) into this relation and then substituting that result into (3) and integrating (step four) gives

$$\begin{aligned}(H^n_{z,i,j,k} - H^n_{z,i+1,j,k})\delta z + (H^n_{x,i+1,j,k+1} - H^n_{x,i+1,j,k})\delta x \\ - \frac{V^n_S + \sum_{m\neq j} E^n_{y,i+1,m,k}\delta y}{R_S} \\ - \frac{(E^{n+1}_{y,i+1,j,k} + E^n_{y,i+1,j,k})\delta y}{2R_s} \\ = \frac{\sigma(E^{n+1}_{y,i+1,j,k} + E^n_{y,i+1,j,k})\delta x\delta z}{2} \\ + \frac{\varepsilon(E^{n+1}_{y,i+1,j,k} - E^n_{y,i+1,j,k})\delta x\delta z}{\delta t} \quad (5)\end{aligned}$$

where δt is the time increment, $\delta x, \delta y, \delta z$ are the cell lengths, time derivatives have been approximated by a forward finite difference, and the $E_{y,i+1,j,k}$ terms have been time averaged, but not the other E_y terms. The first term on the right-hand side is the time-averaged term obtained from $\boldsymbol{J} = \sigma\boldsymbol{E}$, and the second term is derived from the $\partial\boldsymbol{E}/\partial t$ term. Solving for the new value of E_y from (5) gives (6), shown at the bottom of the page. Relations similar to (6) for other circuits may be readily derived; for example, in circuits for which the circuit equations relate V_c and the time derivative of I_c, the time derivative can be used to obtain the new value of I_c, which can then be used directly with the discrete form of (3) instead of (6). Depending on the relation of I_c to V_c, various time-averaging options may be needed to avoid instabilities, as described and illustrated in [9]. Standard relations for E^{n+1} apply for cells not connected to lumped-element circuits.

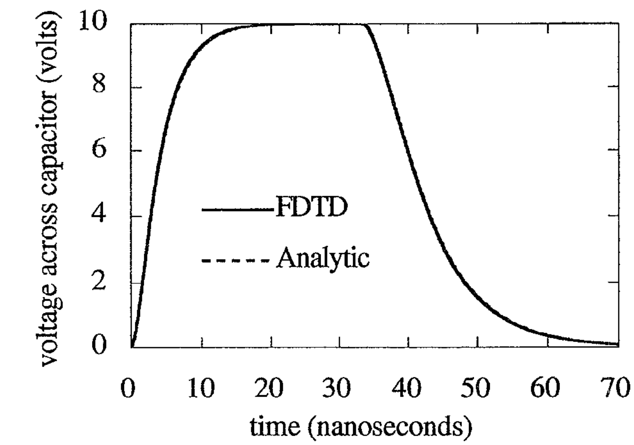

Fig. 2. Voltage across a two-dielectric ($\varepsilon_{r1} = 10, \varepsilon_{r2} = 30$) parallel-plate capacitor excited by a voltage source V_s in series with a resistor R_s. For $0 < t < 32$ ns, $R_s = 2\,\text{k}\Omega$, and $V_s = 10(1 - \exp(-3t/\tau))$ V. For $t > 32$ ns, $R_s = 4\,\text{k}\Omega$, and $V_s = 10\exp(-2t/\tau)$ V, where τ is the system RC time constant. The capacitor was 10×10 cells in area with eight cells between the plates (four cells with $\varepsilon_{r1} = 10$. and four cells with $\varepsilon_{r2} = 30$); the cubical cells were 1 mm on a side, and $\delta t = 1.667$ ps. The FDTD and analytic results are nearly indistinguishable.

III. Validation and Illustration

We validated the formulation illustrated by (6) by comparing FDTD calculations for a number of circuits with analytical results. Fig. 2 shows, for example, very close agreement between the calculated and analytical voltages across a two-dielectric parallel-plate capacitor (no fringing fields) excited by a voltage source in series with a resistance. Also, the FDTD calculated electric field was within 0.05% of the analytical results. This validation is very significant because the FDTD E fields are properly discontinuous across the dielectric boundary by the ratio of the permittivities in agreement with the analytical solution, even though the exciting voltage source and series resistor are connected across multiple FDTD cells with different dielectrics, and the voltage across each FDTD cell between the source connections is calculated, not forced, as is often done in FDTD simulations.

We next demonstrate the usefulness of the formulation with a more complex example in which a source and a load extend

$$\begin{aligned}E^{n+1}_{y,i+1,j,k} = &\frac{\dfrac{\varepsilon}{\partial t} - \dfrac{\sigma}{2} - \dfrac{\delta y}{2R_s\delta x\delta z}}{\dfrac{\varepsilon}{\delta t} + \dfrac{\sigma}{2} + \dfrac{\delta y}{2R_s\delta x\delta z}} E^n_{y,i+1,j,k} - \frac{\left(\dfrac{V^n_s}{\delta y} + \sum_{m\neq j} E^n_{y,i+1,m,k}\right)\delta y}{\left(\dfrac{\varepsilon}{\delta t} + \dfrac{\sigma}{2} + \dfrac{\delta y}{2R_s\delta x\delta z}\right)R_s\delta x\delta z} \\ &+ \frac{(H^n_{z,i,j,k} - H^n_{z,i+1,j,k})\delta z + (H^n_{x,i+1,j,k+1} - H^n_{x,i+1,j,k})\delta x}{\left(\dfrac{\varepsilon}{\delta t} + \dfrac{\sigma}{2} + \dfrac{\delta y}{2R_s\delta x\delta z}\right)\delta x\delta z}. \quad (6)\end{aligned}$$

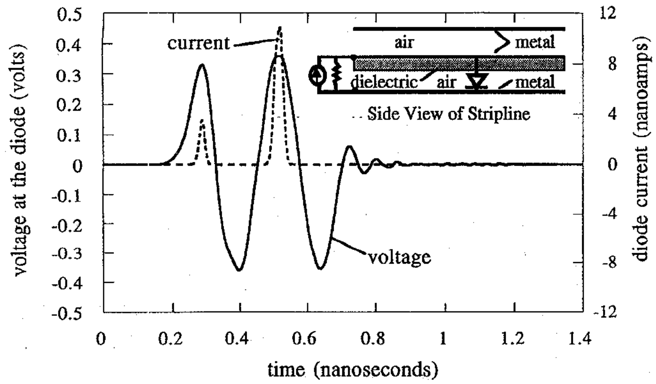

Fig. 3. Voltage across and current through a diode located 4 cm from the input of a shielded suspended microstrip line excited by a current source in parallel with a 500 Ω resistance. The current source generates just two cycles of a 4-GHz sine wave. The dielectric has a relative permittivity of 8 and conductivity of 0.02 S/m. The current shows exponential sensitivity to the voltage.

across multiple FDTD cells, which requires a relationship like (6). The configuration is a shielded suspended microstrip line excited with a current source in parallel with a resistance, with a diode load. The current source generates two cycles of a 4-GHz sine wave. To reduce transients, the source is turned on and off with a raised cosine function. The top and bottom conductors of the microstrip model are 20 cells wide, 800 cells long, and 10 cells apart. The center conductor is five cells wide. The bottom air gap is two cells high, the dielectric is two cells high, and the top air gap is six cells high. All cells are cubical cells 0.5 mm on a side. Magnetic walls are located on the sides and ends. The peak value of the current source is 5 mA, the source resistance is 500 Ω, and the FDTD time increment is 0.417 ps. Fig. 3 shows the voltage pulse at the diode and the current through the diode. Dispersion occurs because the TEM mode cannot exist on the microstrip line. We also found that eliminating the air gap between the dielectric and the bottom ground plane in the microstrip line reduces the dispersion. This example, in which the source is connected across inhomogeneously dielectric-filled cells, illustrates the usefulness of the technique developed herein.

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